PRIMARY SCHOOL PRE-SERVICE TEACHERS' SUBJECT MATTER KNOWLEDGE OF MATHEMATICS FOR TEACHING MULTIPLICATION AND ADDITION OF FRACTIONS

By

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Abstract

Many researchers and education stakeholders in South Africa point to the need to develop teachers' mathematical knowledge of the mathematics concepts that they teach to their learners. In this research the researchers explore the understanding of 60 pre-service primary mathematics teachers on multiplication and addition of fractions. Data were generated from the written responses to an assessment as well as semi-structured interviews. The written responses and interviews were analysed using content analysis. The purpose was to explore the methods used by pre-service primary mathematics teachers to solve a given task on multiplication and addition of fraction. Emphasis was also based on whether the pre-service primary mathematics teachers were able to use different methods to solve the given task. The results showed that 46 participants were able to solve the task using one method, while 20 of them were able to use two different methods. The most common method used was based on the part-whole relationship method. Few students identified different other methods of solving the task correctly but some of them were unable to use any method. These results indicate that these students are not yet ready to teach these primary school level concepts even though they have studied advanced mathematics topics as part of their pre-service training. The study recommends that pre-service primary mathematics teachers should also be provided with more structured opportunities to help develop pedagogic content knowledge of the primary school level content as part of their teacher *training programme as well as possess a deep understanding of different interpretations of fractions.*

Keywords: Exploratory study, multiplication and addition of fraction, primary school pre-service teachers, subject matter knowledge of mathematics

Introduction

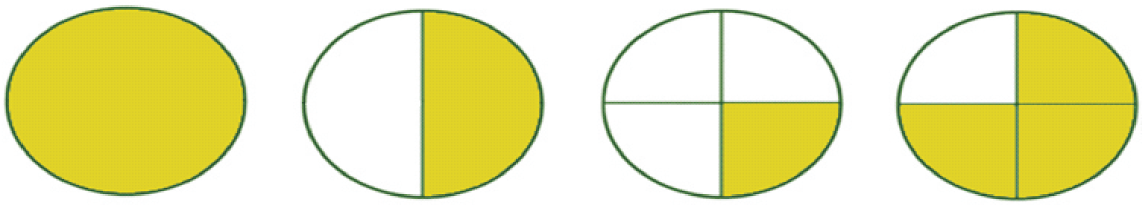
This research explores primary school pre-service teachers' knowledge of mathematics for teaching multiplication and addition of fractions in primary schools. The research is motivated by South African learners' poor performance in mathematics as evident in the Annual National Assessment findings and the Grade 12 moderators' report (Department of Basic Education, 2015). The report suggests the need for a trivial examination of the facilitators of teaching and learning of mathematics, especially at primary school level. This research will use participants responses to a semester examination for graduating students which was followed by semi-structured interviews. The course that the students took before the semester examination was intended to help deepen their understanding of basic numeracy including aspects such as operations on fractions with respect to meaning and use of representation. In most developing economies mathematics educators, researchers and other stakeholders raise concerns about the quality and effectiveness of mathematics learning outcomes. South Africa, for instance, teachers who do not have a sufficiently robust knowledge of primary mathematics are recruited into the teaching profession because of the large demand for qualified teachers (Bowie & Reed, 2016; Deacon 2016; Ndlovu, Amin, & Samuel, 2017). Hence many teacher education institutions are struggling to break the cycle of mediocrity, where school leavers who were poorly taught as a learner are returned to the schools as poorly prepared teachers (Deacon 2016). This research is situated in the context of under-prepared primary school pre-service teachers who are struggling themselves with the primary level mathematics that they are expected to teach.

However, changes in the mathematics curriculum for South African schools brought in a more flexible and connected way of looking at primary level mathematics like fraction as the curriculum includes developing the strong conceptual understanding that learners must have (Rey, Lindquist, Lambdin, Smith, Rogers, Falle, Frids & Benneth 2012). Fractions require much attention because they present a hurdle as learners attempt to transfer their understanding of whole numbers to a new but related class of numbers (Chinnappan & Forrester 2014; Siegler Fazio, Bailey & Zhou 2013; Ubah & Bansilal 2018). Children encounter fractions and fraction-related concepts both in real-life and classroom situations. A sound understanding of what fractions are would help children make sense of a multitude of other ideas in their daily life. Regardless of the context in which children engage fractions, it is generally agreed that fractions provide teachers with insight into developments in children's understanding of numbers and relations among numbers. These understanding are built on children's personal experiences, intuitions, and formal knowledge gained in the classroom.

The concept of fraction is an important idea that permeate much of the mathematics studied in the general education and training (GET) band in South African schools. Many research studies argue that teachers' understanding of the mathematics concepts should go beyond what is taught to their learners, because they must respond to demands different from just being able to solve a problem (Ndlovu, Amin & Samuel, 2017; Ubah & Bansilal 2018). This demand is associated with the pedagogical content knowledge (PCK) of fraction concept, for example, primary school teachers need to understand that comparing, representation and ordering fractions allows learners to develop a sense of fractions as quantity, as well as the size of a fraction, both are necessary prior knowledge components for understanding fraction operations (Bruce, Cheng & Flynn 2013). The purpose of this research was to explore the methods used by primary school pre-service teachers to derive the solution of a task on multiplication and addition of fractions using different methods.

The Concept of Fraction

Fraction represents part of a whole. When an object is divided into several parts, the fraction shows how many of those parts you have. Sometimes the best way to learn about fractions is through a picture. The pictures below show how the whole of a circle is broken up into different fractions. The first picture from the left shows the whole, and the other pictures shows fractions of that whole (see figure 1).



1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$

Figure 1. Fractions of a whole

Basturk (2016) has identified that the meaning of fraction includes part-whole, measurement, division etc. Proper understanding of fractions needs exploring all these different meanings. Whole-number knowledge is overgeneralized by students. Unfortunately, fractions are generally reduced to only a meaning part-whole by both textbook writers and teachers. As already mentioned, there are many meaning of fractions. Focusing on only one of them is not enough to understand fractions completely. Thus, the researchers such as Hackenberg and Lee (2015) and

Hansen, Dews, Dudgeon, Lawton and Surtees (2017) argue that students would better understand fractions with other meanings; part-whole, division, measurement, operator and ratio.

Part-whole is the most known and used meaning as it involves a whole divided into equal parts and we select or take some of them. Sure, this whole can be a group of people or a length or pizza. The meaning of division in fraction is rather encountered in the situations of sharing. There is equally sharing some quantities between some people or things such as sharing 20 pencils with 5 learners. Sometimes, fractions are used to identify a length or a measurement piece to determine the length of an object. This meaning refers that fraction represent measurement of quantities such as length, area, weight or volume which are unable to be represented with whole numbers. The meaning of fraction as operator refers to enlarge or reduce a certain quantity. Definition of fraction based on operator can help to understand the multiplication process of fractions (Hackenberg & Lee, 2015). The ratio is another meaning of fractions. For instance, can be the probability of an event being two in five. Teaching and learning of fraction is a complex process, but the most important factor that can influence learners understanding is the teachers' mathematics knowledge for teaching fraction.

Tobias, Olanoff& Lo, 2012 examined pre-service teachers mathematical content knowledge (MCK); the study revealed that this MCK may be insufficient for effective teaching of multiplication and division with fractions. In view of this, Van Steenbrugge, Lesage, Valcke, &Desoete (2014) study on preservice teachers' knowledge of fractions revealed that fractions are notoriously difficult for learners to learn and for teachers to teach. Whitehead and Walkowiak (2017) examined preservice elementary teachers' change in their understanding of fraction operations while taking a mathematics methods course focused on grades 3-5. Using a paired t-tests, the findings revealed a statistically significant improvement on most items on the assessment and on the total test score.

In line with this, Olanoff, Lo and Tobias (2014) presents a research summary of prospective elementary teachers' mathematical content knowledge in fractions. Across the time frames, the trend in the research has moved from looking almost entirely at prospective teachers' understanding of fraction operations, to a more balanced study of both their knowledge by operations and fraction concepts. In addition, Ubah and Bansilal (2018) study revealed that many of the pre-service teachers coped well with addition and subtraction of common fractions with the same denominator. However, more than 52% struggled to carry out these operations on common fractions with different denominators, showing that their conceptions had not developed into object-level structures.

Basturk (2016) study showed that most of the student teachers understood the introduction to fractions as closely related to this meaning such as dividing a cake and pizza into equals parts or shading a region. Chinnappan and Forrester (2014) distinguish between procedural and conceptual knowledge of fractions by pre-service teachers when examining the impact of an instructional model designed to improve the conceptual understanding of fraction concepts and operations. In addition, teachers should develop a deep understanding of the different interpretations of fractions (Hansel, et al. 2017). Chinnappan and Forrester (2014) recommend that pre-service teachers can be supported, within an existing teacher education programme, in constructing conceptually and procedurally robust content knowledge through the development of appropriate meaning, representations of fraction concepts and use of basic operations.

Statement of problem

The review of the literature revealed insufficient and nonconclusive research on pre-service teachers' understanding of multiplication and addition of fractions. The reviewed literature indicated that learners at all levels of education have difficulties in representation and operations of fraction. Moreover, it has been a challenging task for teachers to help their learners understand the connections between different representations of fraction in a flexible way. Hence there is a need to explore the understanding of primary school pre-service teachers in this area. This research provides an exploratory framework that can be used to find out more about how the participants of this study solve a task on multiplication and addition of fractions.

Research Questions

This research intends to respond to the following research questions:

1. What methods do primary school pre-service teachers use to solve a given task on multiplication and addition of fractions?
2. To what extent does the primary school pre-service teachers use different methods to solve a given task on multiplication and addition of fractions?

Research Methodology

Research Design

A qualitative research method in general and case study approach were used to find out primary school pre-service teachers' knowledge in solving a task on multiplication and addition of fractions. Qualitative research methods permit an in-depth investigation of single or small number of units at a point (over a period) in time (Hsieh & Shannon 2005).

Selecting Participants

The exploratory researchwas carried out with 60 primary school pre-service teachers (purposively sampled) enrolled in foundational course in mathematics forming part of their bachelor's in education (B.Ed.) degree at a South African University, because they did not pass mathematics at Grade 12 level. This course was intended to help deepen their understanding of basic numeracy including aspects such as operations on fractions with respect to meaning and use of representation. In this research, five participants were conveniently sampled for the semi-structured interview. The purpose is to select information-rich cases whose study will illuminate the questions under study (Ratcliff, 2016).

Data Collection

Data to gauge primary school pre-service teachers' knowledge of fraction permeate from multiplication and addition of fractions was obtained from the participants' responses to a semester examination for graduating students, which was followed by semi-structured interviews. The semester examination test items on Fraction and its operations was conducted at the end of the semester after exposing the participants to concepts on fraction through the semester. However, the test items were adopted from the previous question on same course over the years, hence should have been validated and its reliability tested before the adoption by the author. A written task based on multiplication and addition of fractions was designed for 60 primary school pre-service teachers and used for this study to probe their knowledge of fraction. The participants written responses were analysed and thereafter five primary school pre-service teachers conveniently sampled participated in a semi-structured interview. The students were probed generally about their knowledge of fractions and more specifically about their responses to the written task.

Data Analysis

Analysis of data entails breaking down the information gathered into elements to obtain responses to research questions (Sauro, 2015). In this research, the qualitative data from written responses and interviews were analysed using content analysis. The interview data were organized to get an overview of what it revealed, and test responses were grouped into manageable themes (Ratcliff, 2012). For the detail of the task, see Table 1 and Figure 2.

Task.

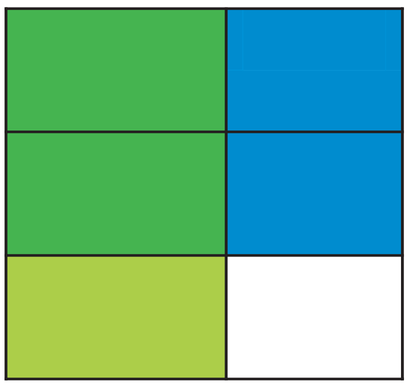
1. **Using two different methods, simplify $\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{5}$ Leave your answer as a fraction. Calculators may not be used to find answers. Show all your workings:**

Figure 2. Written task

Expected Responses to the Written task

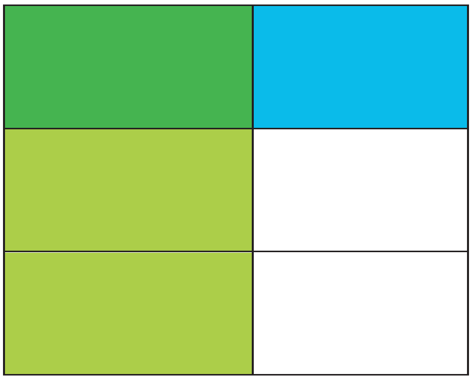
There are different methods of solving the given task based on multiplication and addition of fractions. The pre-service primary mathematics teachers were expected to use any two different methods to solve the task. The methods identified from the participants written paper are used according to BODMAS rule.

Table 1. Possible Responses

Methods	Possible Solution
For multiplication and addition of fraction.	Solve: $\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{5}$ Using BODMAS rule $(\frac{1}{2}\times\frac{2}{3})+(\frac{1}{2}\times\frac{1}{5})$
First: Visual & Bowtie Method	Solve: $\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{5}$ First solve $\frac{1}{2}\times\frac{2}{3}$ $\frac{1}{2}\frac{1}{2}$  $\frac{1}{3}$ each (row) The thick shaded green part is the point of intersection (numerator) while the total squares of '6' is the denominator. Hence, $\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}$ ……….. **(i)**

Then $\frac{1}{2} \times \frac{1}{5} = ?$

$\frac{1}{2}\frac{1}{2}$



$\frac{1}{5}$ each square (row)

The thick shaded green part is the point of intersection (numerator) while the total squares is the denominator (10 squares)

Hence,

$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$ ……….. **(ii)**

Adding (i) and (ii) together we have:

$\frac{2}{6} + \frac{1}{10} = ?$

Adding any of the equations (i) and (ii) we have;

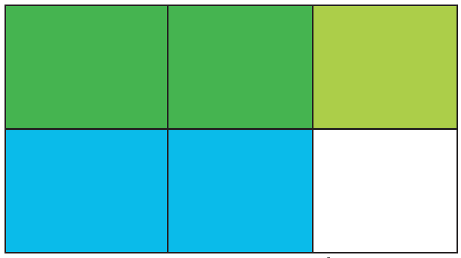
$\frac{2}{6} + \frac{1}{10}$

$$= \frac{(2 \times 10)+(6 \times 1)}{6 \times 10} = \frac{20+6}{60} = \frac{26}{60}$$

Simplifying we have $\frac{26 \div 2}{60 \div 2} = \frac{13}{30}$

<table>
<tr>
<td>Second: Fraction chart & Bowtie method</td>
<td>
Solve: $\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{5}$

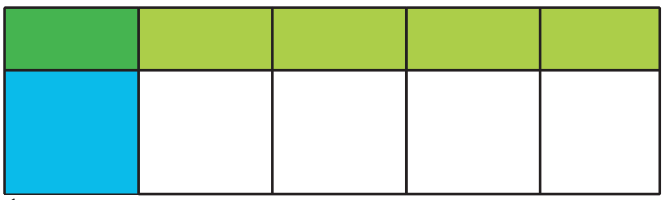
First solve $\frac{1}{2} \times \frac{2}{3}$

$\frac{2}{3}$

$\frac{1}{2}$

You divide a square into three parts (row) and shade two parts out of the three (light blue). Further, divide the three parts into two (light green) i.e. half of $\frac{2}{3}$ to get the dark shaded part (dark green). The part shaded dark green forms the numerator (2) while the total number of squares is the denominator (6).

That is $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$ ……………………… **(i)**

Then $\frac{1}{2} \times \frac{1}{5}$ =?

$\frac{1}{5}$

$\frac{1}{2}$

You divide a square into five parts (row) and shade one part out of the five (light blue). Further, divide the five parts into two (light green) i.e. half of $\frac{1}{5}$ to get the dark shaded part (dark green). The part shaded dark green forms the numerator (1) wh ile the total number of squares is the denominator (10).
</td>
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</table>

	That is $\frac{1}{2} \times \frac{2}{3} = \frac{1}{10}$ ………………………. **(ii)** Adding (i) and (ii) together we have: $\frac{2}{6} + \frac{1}{10} = ?$ Adding any of the equations (i) and (ii) we have; $\frac{2}{6} + \frac{1}{10}$ $= \frac{(2 \times 10)+(6 \times 1)}{6 \times 10} = \frac{20+6}{60} = \frac{26}{60}$ Simplifying we have $\frac{26 \div 2}{60 \div 2} = \frac{13}{30}$
Third: Part -whole method (Traditional method)	Solve: $\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{5}$ First solve $\frac{1}{2} \times \frac{2}{3} =?$ $\frac{1}{2} \times \frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6}$ ……………………...(i) Next $\frac{1}{2} \times \frac{1}{5} = \frac{1 \times 1}{2 \times 5} = \frac{1}{10}$ ………………(ii) Adding any of the equations (i) and (ii) we have; $\frac{2}{6} + \frac{1}{10}$ $= \frac{(2 \times 10)+(6 \times 1)}{6 \times 10} = \frac{20+6}{60} = \frac{26}{60}$ Simplifying we have $\frac{26 \div 2}{60 \div 2} = \frac{13}{30}$ OR Adding any of the equations (i) and (ii) we have; $\frac{2}{6} + \frac{1}{10} =$ Find the L. C. M of the denominators is 6 $\times 10$ = 60 then that becomes the new denominator; $= \frac{(60 \div 6) \times 2 + (60 \div 10) \times 1}{60} = \frac{20+6}{60} = \frac{26}{60}$ By simplification $\frac{26}{60} = \frac{26 \div 2}{60 \div 2} = \frac{13}{30}$

Research Findings

Overall summary of the results of the written responses

There were three different methods identified from the participants written scripts that they used to solve the given task on multiplication and addition of fractions as explained in Table 2 and are referred to as the Visual and Bowtie method (VB), the Fraction chart and Bowtie method (FB) and Traditional method (TM). The researcher looked at the different methods chosen by the participants and whether the solution they obtained was correct, at least one correct step or no correct response, for each of the methods they chose. Table 2 presents the results for all these categories:

Table 2. Results based on methods for solving a task on multiplication and addition of fractions

Summary of	*responses*			
Description of methods	Frequency and percentage of Correct response	Number and percentage Showing at least one correct step (multiplication of addition of fraction) but incorrect final answer	Frequency and percentage of no correct response	Frequency and percentage of no response
Visual & Bowtie method	4(7%)	-	-	56(93%)
Fraction chart & Bowtie method	2(3%)	-	-	58(97%)
Part-whole method (Traditional method)	42(70%)	8(13%)	2(3%)	8(13%)

As indicated in table 2, 42 of the participants worked out the task correctly using part-whole method, representing 70% of the group, 8(13%) that used part-whole method had only one step correct (multiplication or addition) but incorrect final answer while 2(3%) of the participant had wrong step and answer using same part-whole method. Out the 60 participants, 4(7%) used visual and bowie method to work out the given task correctly while 2(3%) of the participants worked out the task correctly using fraction chart and bowtie method. It is important to note that the participants that used visual & bowtie and fraction chart & bowtie used the method correctly without any error. More than half (80%) of the participants were able to use at least one method to solve for multiplication and addition of fraction. However, this number dropped by more than half to 14(23%) when it came to use two different methods to solve the problem correctly (See table 3).

Table 3. Results based on two methods of solving the fraction task

Summary of responses			
Description of methods	Correct response	Shows at least one correct step (multiplication of addition of fraction) but incorrect final answer	No correct response
Visual & Bowtie method /Part -whole method	8(13%)	-	-
Fraction chart & Bowtie method/ part-whole method	6(10%)	-	-

As seen in Table 3, 8(13%) out of the 60 participants that identified the visual/bowtie method and part-whole method and used them correctly to solve for the given task on fraction. 6(10%) out the 60 participants identified the use of fraction chart /bowtie method and part-whole method and used them correctly to solve the task on multiplication and addition of fraction. An indication that fewer participants 14(23%) of the participants could use varied methods to find the solution to the task on multiplication and addition of fractions.

Analysis of individual written responses to the task and corresponding interview

When one of the participants (Lindo) was probed during the interview about his performance in the written task, he replied that he "was blank" but attempted the given task and the solution is wrong. See figure 3 for Lindo's written response.

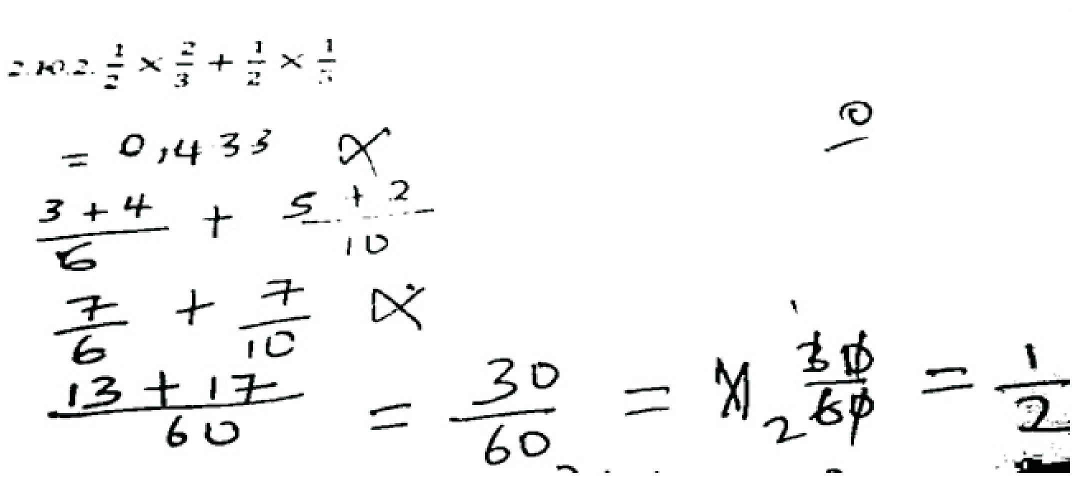

Figure 3. No correct written response of Lindo

Lindo's approach is an example of no correct response to the given task. He started by finding the LCD in multiplication of fractions is an evidence of poor knowledge of basic fraction operations. After finding the LCD, Lindo applied a 'cross- multiplication' on the two fractions and added the results to get the numerator. Then, in the addition of $^7/_6 + {}^7/_{10}$. Lindo started first by identifying the denominator of the sum as 60, then 'cross adding' each fraction, and adding the result to get the numerator.Another participant (Thobibe), written response is an example of one correct step but incorrect final answer (see figure 4). that had a dialogue between the researcher (R) and himself appears in Box 1.

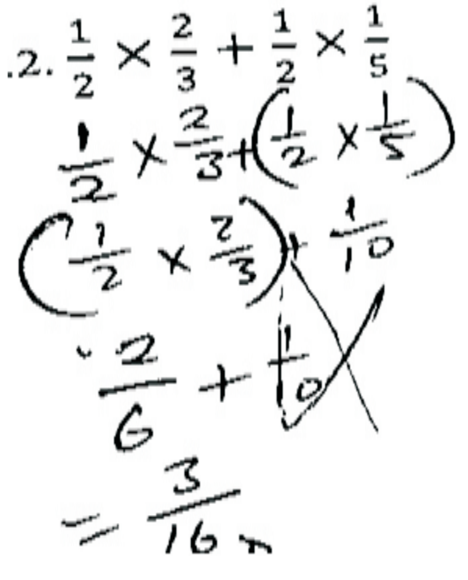

Figure 4. Written response of Thobibe

Thobibe used correct traditional method (part-whole) of multiplication but could not use appropriate method to add the two fractions, hence an incorrect final answer to the task. Here is the dialogue between researcher (R) and Thobibe (T) appearing in Box 1.

Line 1. R: About the written task, that is, $\frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{5}$ is $\frac{3}{16}$? How did you get it?

Line 2 . T: mmmmm, I think with BODMAS rule am going to start with multiplications first

Step 1: $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$

Step 2: $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

Step 3: $\frac{2}{6} + \frac{1}{10} = \frac{3}{16}$ that is the answer

Line 3. R: How did you get $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$?

Line 4. T: The first fraction will multiply the second fraction. There is no lowest common denominator because it not addition.

Line 5. R: Tell me the process you followed to get your answer in step 1?

Line 6. T: I multiplied the numerator of the first fraction to the numerator of the second fraction to get 2, then the denominat or of the first fraction to the denominator of the second fraction to get 6, hence I have $\frac{2}{6}$. I followed the same process in step 2.

Line 7. R: What of the third step?

Line 8. T: Step 3 has to do with addition; I followed the same process but used addition because am asked to add the two fractions.

Line 9. R: So, in addition and multiplication you work on the numerators and denominators separately using the given operation?

Line 10. T: Yes ma that is the rule.

Box 1. Extracts of interview with Thobibe

However, the written response of Khan shown on figure 5 is another one correct step and incorrect final answer using part-whole method (traditional method). See figure 5.

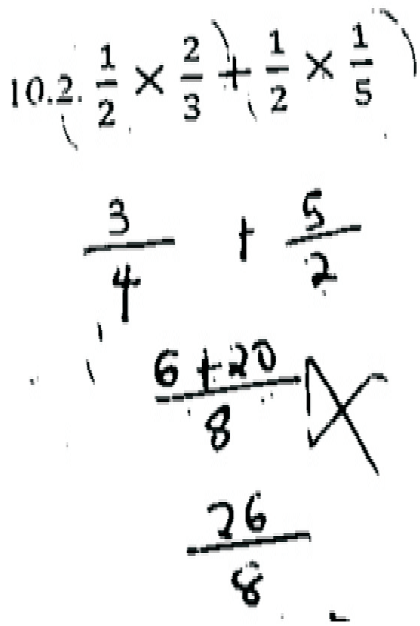

Figure 5. Khan no correct response to the written task

From Khan written response, the process of multiplying the two different fractions were wrong (cross multiplication). The participants assume the sign of multiplication to be cross multiplication; where he multiplied the numerator of the first fraction to the denominator of the second fraction to produce wrong answer. In the case of addition; he applied Bowtie method correctly but derive an incorrect final answer to the task because of the wrong multiplication process done. An interview with Khan provides much insight into her reasoning about multiplication and addition of fractions with different denominators. See interview extract between Khan (K) and the researcher (R) in Box 2.

In the following extract, R stands for the researcher and K stands for Khan:

Line 1. R: what do you remember about fractions?

Line 2. K: fraction is basically about knowledge of division of an object. For example, we are five in a room, and we have one orange we need to share, so we cut it in pieces so that it will accommodate all of us.

Line 3. R: Must the cutting be equal?

Line 4. K: Not exactly, it depends on choice as there are different types of fraction.

Line 5. R: Alright: Can you mention the different types of fraction we have?

Line 6. K: The mixed, the common, the improper.

Line 7. R: Which one is the mixed?

Line 8. K: Examples of mixed fraction are 2 ½, 27 ¼, 10 $\frac{5}{7}$ and others.

Line 9. R: About the written task, that is, ½ × $\frac{2}{3}$ + ½ × $\frac{1}{5}$ is $\frac{26}{8}$? How did you get the answer?

Line 2. K: using BODMAS rule am going to start with multiplications first

Step 1: $\frac{1}{2} \times \frac{2}{3} = \frac{3}{4}$

Step 2: $\frac{1}{2} \times \frac{1}{5} = \frac{5}{2}$

Step 3: $\frac{3}{4} + \frac{5}{2} = \frac{26}{8}$ that is the answer.

Line 3. R: How did you get $\frac{1}{2} \times \frac{2}{3} = \frac{3}{4}$?

Line 4. K: This sign in between the two fractions shows cross multipl ication. Because of that, I will first multiply 1 to 3 to get 3 and 2 multiply by 2 to get 4; then I have fraction $\frac{3}{4}$. The same to the second two fractions.

Line 5. R: You mean your process of multiplication is; numerator of the first fraction multiplied by the denominator of the second fraction and vice versa?

Line 6: K: Yes ma'm.

Line 7: R: Alright; in the case of addition of two fractions what do you do?

Line 8. K: For addition of two fractions, you must find the lowest common denominator $\frac{3}{4} + \frac{5}{2}$ is 4 × 2 = 8 i.e. the LCD is 8. Then I will also multiply the numerator of the first fraction to the denominator of the second fraction, then multiply the denominator of the first fraction to the numerator of the second fraction and finally add up the two answers to get the new numerator because I am asked to add.

Line 9. R: So, in addition and multiplication you multiply the numerators and denominators together using the given operation?

Line 10 . K: Yes ma.

Line 31. R: Suppose you are given $\frac{a}{b} \times \frac{c}{d}$ how will you solve it to get the answer?

Line 32. K: It is now complicated but let me try. "a' multiply by 'd' = 'ad'; then 'b' multiplies by 'c' = 'bc'. The answer will be $\frac{ad}{bc}$.

Box 2. Interview with Khan

Khan's understanding of fraction itself seems to contain many contradictory ideas. She sees a fraction arising from a whole that is divided into parts (cutting out an orange into five parts), which may not necessarily be equal in size (Box 2, Line 2). The size depends on a person's choice because the different kinds of factions depend on whether you have equal parts or not. Further insight about different kinds of fractions is provided by her categories of mixed, common and improper, suggesting that these are different types (Line 6). Smith in carrying out his operations on fractions clearly views the fraction as two whole numbers separated by a line.

Another type of one correct step but incorrect final answer is shown by Shabalala; involved adding the same whole number to the numerator and the denominator. There is a mutation of the rules used to create equivalent fractions by multiplying a fraction by a factor and dividing it by the same factor. However, in this case the student added the number to the numerator and denominators (See figure 6).

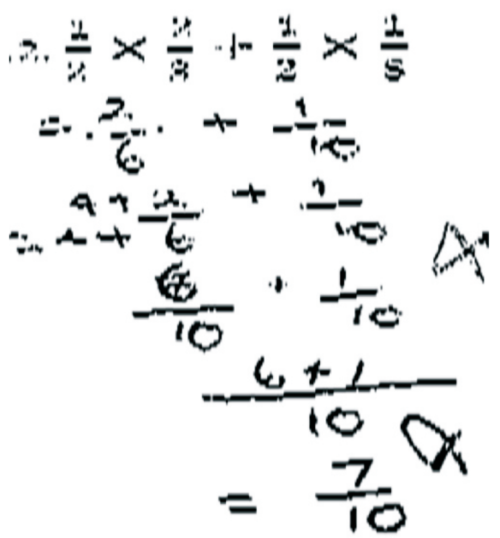

Figure 6. Shabalala written response

Shabalala attempts to create equivalent fractions by adding the same number to the numerator and the denominator. In Shabalala's written response, the first step of multiplication of fraction followed the correct procedure but in next step which was addition of fraction, Shabalala, in an attempt to make the denominators of the two fractions equal, used his own rule for equivalence by adding the same number (4) to the numerator, and denominator of the first fraction in an attempt to create a fraction with the same denominator (10) as the second fraction. His third step is the correct procedure of adding up fractions was used to find the supposed answer. He also displayed poor knowledge of fraction operations by adding 4 to the numerator and denominator to get a fraction with denominator of 10. An interview with Shabalala provides much insight into her reasoning about multiplication and addition of fractions with different denominators. When asked during the interview, why he could not engage constructively on the question, Shabalala said he did not know such question will be asked and has forgotten the process of using the different methods to solve a task on multiplication and addition of fractions.It is hard to imagine how this concept will be taught in his learners, if he does not receive any further instruction on the teaching of the concept.

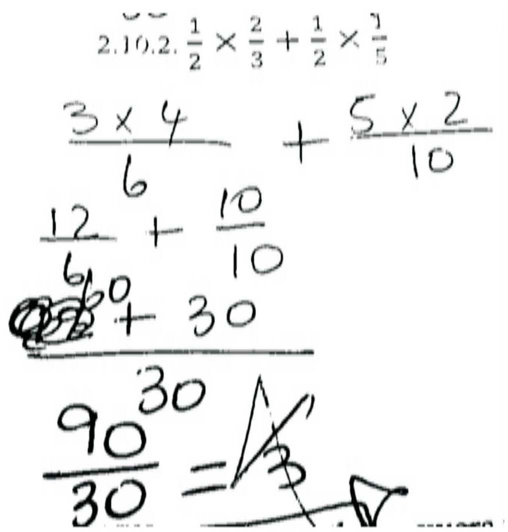

Figure 7. Zanele incorrect written response

Zanele's response in Figure 7, follows the order of multiplication before addition according to the BODMAS rule. She has attempted to carry out multiplication of fractions in the first step, on two pairs of fractions, using the incorrect method consistently. For example, in the product ½ ×$^2/_3$, she took the LCD of the two fractions (6) as the denominator. She then 'cross multiplied' and added the results ((1×3) + (2×2)) to generate the numerator. This method was applied consistently for both sets of fractions being multiplied in the first step. Having generated a fraction as an answer to each product she then proceeded to add the two fractions correctly. Zanele interview extract is shown in Box 3.

Line 1 .R: You solved $\frac{1}{2}\times\frac{2}{3}$ to get $\frac{12}{6}$, and ½ $\times\frac{1}{5}=\frac{10}{10}$. Explain how you derive the answers?

Line 2. Z : First, I derived the LCD of each set of fractions by multiplying the two denominators in each case. Using the multiplication sign, I multiplied across and multiplied the answers together to get the numerator. The same is applied to the second set of fractions. After that, I added up the answers I got from multiplication. [*While explaining, Zanele demonstrated with her pen the movement from the numerator oft the first fraction to the denominator of the second in the opposite direction*]

Line 3 .R: Suppose t hat a learner expresses the rule multiplication of fractions like this: $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$; Is the learners response correct and if not identify the error?

Line 4. Z: The response is not correct. The learner was using the

$\frac{adcb}{bd}$ addition rule but the correct response should be (a $\times d$) $\times$ (c$\times$ b) $\div$ (b $\times d$) =

Box 3: Extracts of the interview with Zanele on Multiplication of fractions

Zanele's follow up explanation (line 4, Box4) above shows how deeply ingrained the 'cross multiplication' technique is in her schema. She has been able to use a general algebraic form to explain how the rule is applied. Her response also alludes to an "addition rule" which involves an application of the 'cross multiplication' rule in adding and subtracting fractions. In view of the findings, it is hard to imagine how this fraction concept will be taught in the classroom, if these pre-service primary mathematics teachers do not receive any further instruction on the teaching of the fraction concept before starting their teaching profession. However, there were some differences observed between male and female participants, the issue of gender is beyond the scope of this research.

Discussion

The research showed that most common method chosen by the participants, in solving the task on multiplication and addition of fraction was the traditional method (par-whole method). This method required the use of BODMAS rule to multiply the two sets of fractions first; this is done by multiplying the numerators together and the denominators together to get an answer. Afterwards, the two answers derived from multiplication will be added either using lowest common denominator process or conversion of the fractions to equivalence fraction. This confirms the findings of Basturk (2016) who observed that when saying fractions, first thing which comes in the student teachers' mind is the meaning "part-whole" of fractions. However, it is a little different to the finding of Simon (2018) where prospective teachers have limited part-whole concept knowledge of fraction.

Similarly, in this research, pre-service primary mathematics teachers struggled with identification and use of other methods to solve the given. This finding agrees with (Siegler et al. 2013) study that revealed that the key construct that supports the learning of fractions is the part-whole relationship. The study by Charalambous and Pitta-Pantai (2005) observe that the method of teaching fractions as part of a whole was necessary but that it was not appropriate to use this as the only way to teach fractions. In view of this, the researcher suggest that teachers should develop a deep understanding of the different interpretations of fractions. However, Tall, Lima, & Healy, (2014) observed that when learners have gaps in knowledge and skills that should have been developed earlier on, conceptual development of more complicated concepts is severely retarded.

The findings of this research revealed that 80% of the participants were able to correctly work out the given task on fraction using one method. In terms of using two different methods, only 23% successfully applied two different methods to solve same given task on multiplication and addition of fraction. This raises an important issue about the kind of knowledge that teachers need. Shulman seminal description of pedagogic content knowledge (PCK) described this submission as; subject matter knowledge for teaching and the ways of representing the subject to make it comprehensible to others. PCK includes an understanding of what makes the learning of specific topics easy or difficult and the conceptions or preconceptions that students of different levels bring with them to the learning environment (Shulman, 1986). Being able to produce the correct answers to questions based on school level content is just one small part of a mathematics teacher's task. Beyond that, teachers should be able to decide upon sequence of activities to introduce learners to new concepts; to link new content to big ideas in mathematics, to provide clear and unambiguous explanations; to design practice exercises and other consolidation activities as part of the teaching. Moreover, teachers should be able to draw upon links between examples focusing on examples, counter examples, while also bringing in and making links across different representations to deepen their learners understanding of the concept. As research on teachers' pedagogic content knowledge indicates, there is much more than just knowing the common content knowledge (Shulman, 1986; Ball, Thames, & Phelps, 2008). Teachers, who struggle to solve problems based on basic applications of well-known procedures, will be severely hampered in trying to teach their learners. They will not be able to recognise the demands of assessment items and will not be able to design well-structured assessments; neither will they be able to provide appropriate feedback to their learners, because PCK skills are constructed on the foundation of their content knowledge. Thus, the teachers' limited understanding is an impediment to the pedagogic content strategies that they will be able to draw upon in the class.

However, this research showed that some pre-service teachers did not develop the insight that was necessary to make connections between definition, representations and basic operations of fraction. The conditions under which the pre-service primary mathematics teachers learnt fractions may be a factor that has made it difficult for them to acquire the necessary skills. Perhaps learning opportunities that are more conceptually grounded instead of the rule-based method may be more successful and hence programs for pre-service primary mathematics teachers may need to offer such opportunities to pre-service mathematics teachers with poor mathematics backgrounds.

Moreover, this research has provided significant insights to the existing literature by identifying and analysing the pre-service primary mathematics teachers' knowledge of multiplication and addition of fraction. It is important that teachers should develop a deep understanding of different interpretations of fractions (Hansel, et al. 2017). The in-depth interviews provided evidence of how pre-service teachers struggled with making sense of the parameters used in the various forms that fraction can be represented. The results point to the need for pre-service primary mathematics teachers to be given challenging tasks which can help them interrogate their understanding of these fundamental concepts that they will be teaching in future. This is in line with Chinnappan and Forrester (2014) identified that pre-service teachers can be supported, within an existing teacher education programme, in constructing conceptually and procedurally robust content knowledge through the development of appropriate meaning, representations of fraction concepts and basic operations.

Conclusion and Recommendations

This research focused on the written responses of 60 pre-service primary mathematics teachers to an item based on multiplication and addition of fraction. The results showed that although 46 participants were able to solve correctly the given task using one method, most participants were unable to use two different methods to solve the task on multiplication and addition of fraction. The most common method chosen by students was the one based on the traditional method (part-whole method). Some students that identified the visual, fraction chart and use off number line where able to use them correctly to solve the given task. Some of the participants of he research were unable to even solve the task using any method. However, many the pre-service teachers in this study struggled with solving the given task using two methods. These results show that the pre-service teachers have not developed a robust understanding of this primary school level concept that they will be required to teach. This is an urgent problem that needs to be addressed especially because of the large numbers of under-prepared students who are training to become primary school mathematics teachers even though they were not high achievers at school mathematics. It is important that stakeholders need to work together in devising interventions that can be used to offer greater support to those pre-service primary mathematics teachers and on the job teachers who do not have a strong background in primary school mathematics.

As noted in the introductory remarks in this paper, that in some countries such as South Africa, students who do not have a sufficiently robust knowledge of basic mathematics are recruited into the teaching profession because of the large demand for qualified teachers. This raises a concern about whether it is possible for mathematics teacher education programmes to help pre-service primary teachers improve their understanding of the primary school level concepts they will need to teach. Most university programmes focus on developing knowledge of advanced mathematics because of the need for compliance with university accreditation structures. The assumption is that pre-service teachers have developed an understanding of the school mathematics content they need and hence this cannot form the focus of instruction at university. Based on this reality, it is important that pre-service primary mathematics teachers be given opportunities to improve on their knowledge of basic primary mathematics concepts in supportive of a well-structured intervention program. Moreover, newly qualified primary school teachers still need sustained support and mentorship at their schools so that their mathematics knowledge for teaching can be improved and sustained.

References

Alex, J. K., & Mammen, K. J. (2016). Geometrical sense making: Findings of analysis based on the characteristics of the van Hiele theory among a sample of South African Grade 10 learners. Eurasia Journal of Mathematics, Science and Technology Education, 12 (2), 173-188.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389-407.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389-407.

Baştürk, S. (2016). Primary student teachers' perspectives of the teaching of fractions. Acta Didactica Napocensia, 9(1), 35-44.

Bowie, L., & Reed, Y. (2016) How much of what? An analysis of the espoused and enacted mathematics and English curricula for intermediate phase student teachers at five South African universities, Perspectives in Education, 34(1): 102-119.

Bruce, C., Chang, D. & Flynn, T. (2013). Foundations to learning and teaching fractions: addition and subtraction. Submitted to curriculum and assessment branch, Ontario Ministry of Education,3-53.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. Educational Studies in Mathematics, 64: 293-316.

Chinnappan, M., & Forrester, T. (2014). Generating procedural and conceptual knowledge of fractions by pre-service teacher's mathematics education research group of Australasia. Mathematics Education Research Journal, 26,871-896.

Deacon, R. (2016). The Initial Teacher Education Project: Final Report. Johannesburg: JET Education Services.

Department of Basic Education (2015). Annual National Assessment grade 9 Mathematics Test, Mathematics. Pretoria: National Department of Education.

Department of Basic Education (2018). Curriculum and Assessment Policy Statement Grades 10-12: Mathematics. Pretoria: National Department of Education.

Hackenberg, A. J., & Lee, M. Y. (2015). Relationships between students' fractional knowledge and equation writing. Journal for Research in Mathematics Education, 46(2), 196-243.

Hansen, A., Drews, D., Dudgeon, J., Lawton, F. & Surtees, L. (2017). Children's Errors in mathematics England: Learning Matters.

Hsieh, H. & Shannon, S. E. (2005). Three approaches to qualitative content analysis. Qualitative Health Research,15(9), 1277-1288. doi:10.1177/1049732305276687

Kilic, H. (2011). Preservice secondary mathematics teachers' knowledge of students. Turkish Online Journal of Qualitative Inquiry, 2, 17-35.

Leung, I. K. C., & Carbone, R. E. (2013). Pre-service teachers' knowledge about fraction division reflected through problem posing. The Mathematics Educator, 14(1&2), 80-92.

Lovin, L. H., Stevens, A. L., Siegfried, J., Wilkins, J. L. M., & Norton, A. (2018). Pre-K-8 prospective teachers' understanding of fractions: An extension of fractios schemes and operations research. Journal of Mathematics Teacher Education, 21(3), 207-235.

Lowrie, T., & Jorgensen, R. (2016). Pre-service teachers' mathematics content knowledge: Implications for how mathematics is taught in higher education. Teaching mathematics and its applications: An International Journal of IMA, 35(4), 202-215.

NCTM, (2000). Principles and standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Miller, A., Tobias, J., Safak, E., Kirwan, J. V., Enzinger, N., Wickstrom, M., &Baek, J. (2017). Preservice Teachers' Algebraic Reasoning and Symbol Use on a Multistep Fraction Word Problem. Faculty Publications - School of Education. 165. Retrieved fromhttps://digitalcommons.georgefox.edu/soe_faculty/165

Ndlovu, Z., Amin, N., & Samuel, M. A. (2017). Examining pre-service teachers' subject matter knowledge of school mathematics concepts. Journal of Education, (70), 46-72

Olanoff, D., Lo, J., & Tobias, J. M. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus of fractions. The Mathematics Enthusiast, 11(2), Retrieved from http://scholarworks.umt.edu /tme/vol11/iss2/5

Pournara, C., Hodgen, J., Adler, J., & Pillay, V. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in mathematics? South African Journal of Education, 35(3), 1-10.

Ratcliff, D. (2012). 15 Methods of Data Analysis in Qualitative Research Retrieved from http://qualitativeresearch.ratcliffs.net/15methods.pdf.

Reys, R.E., Lindquist, M.M., Lambdin, D.V., Smith, N.L., Rogers, A., Falle, J., Frid, S., & Bennett, S. (2012). Helping children learnmathematics: 1st Australian edition. Milton, QLD: Wiley.

Sauro, J. (2015). Five types of qualitative methods. Denver, Colorado 80206. Retrieved from https://measuring.com/quali-methods.

Shulman, L. (1986). Those who understand knowledge growth in teaching. Educational Researcher, 15(2), 4-14

Siegler, R. S., Fazio, L. K., Bailey, D. H. & Zhou, X. (2013). Fractions: the new frontier for theories of numerical development. Trends in Cognitive Sciences, 17(1), 13-19.

Simon, M.A., (2018). Further elaboration of the learning through activity research program. Journal of Mathematical Behaviour Retrieved from https://doi.org /10.1016/j.jmathb.2018.03.004

Son, J., &Sinclaire, N. (2010). How pre-service teachers interpret and respond to student geometric errors. School Science and Mathematics, 110(1), 31-46.

Tall, D., de Lima, R. N., & Healy, L. (2014). Evolving a three-world framework for solving algebraic equations in the light of what a student has met before. The Journal of Mathematical Behaviour, 34, 1-13.

Taylor .N., & Taylor .S. (2015). Teacher knowledge and professional habits. In N Taylor, S Van de Berg & T Mabogoane (Eds). What makes schools effective? Report of South Africa's national school effectiveness study. Cape Town: Pearson.

Tobias, J. M., Olanoff, D., & Lo, J-J. (2012). A Research Synthesis of Preservice Teachers' Knowledge of Multiplying and Dividing Fractions (pp. 668-673). Proceedings of the 34th Annual Meeting of PME-NA, 2012: Kalamazoo, Michigan: PME-NA.

Ubah, I. J. A., & Bansilal, S. (2018). Pre-service primary mathematics teachers' understanding of fractions: An action-process-object-schema perspective. South African Journal of Childhood Education, 8(2), 539. Retrieved from https://doi.org/10.4102/sajce.v8i2.539

Unlu, M., &Ertekin, E. (2012). Why do pre-service teachers pose multiplication problemsinstead of division problems in fraction? Procedia-Social and Behavioural Sciences, 46, 490-494. Retrieved from https://www.science direct.com/science/article/pii/51877042812012773

Van Steenbrugge, H., Lesage, E., Valcke, M., & Desoete, A. (2014). Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge. Journal of Curriculum Studies, 46(1): 138-161.

Whitehead, A., & Walkowiak, T. A. (2017). Preservice elementary teachers' understanding of operations for fraction multiplication and division. International Journal for Mathematics Teaching and Learning, 18(3), 293-317.